For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. How well Dylan solves puzzles is jointly proportional to the square of the length of his pencil (in cm) and to the square root of the temperature in his room (in $°F$). If Dylan solves 10 puzzles when his pencil is 2 cm long and his room is 108 $°F$, what is the square of the length of his pencil if he solves 5 puzzles and his room is 81 $°F$?

    (A) $\dfrac{10\sqrt{3}}{3}$
    (B) 7
    (C) $\dfrac{4\sqrt{3}}{3}$
    (D) $\dfrac{5\sqrt{3}}{3}$
    (E) NOTA

2. You get bored while studying for this test, and decide to go to Canada. You decide to get a souvenir from your travels, a golden statue costing you 50 CAD(Canadian dollars). The shop gives you a 25% discount, because you are a new customer, but after that raises the price by 10% because they don't like foreigners. Assuming that the exchange rate is 1 USD = 1.5 CAD, how many statues can you buy if you bring 210 USD?

    (A) 7.64
    (B) 8
    (C) 10
    (D) 7
    (E) NOTA

3. Inside the math fortress, the door shuts behind Prabhas, with the only way to escape being through one of 5 doors, each with a different exponent painted: 2^{40}, 3^{30}, 6^{20}, 5^{20}, 7^{15}. Prabhas needs to open the door with the smallest value painted on it to escape the fortress; which exponent does he choose?

    (A) 2^{40}
    (B) 3^{30}
    (C) 6^{20}
    (D) 7^{15}
    (E) NOTA

4. What is the sum of the units digit of 81^{100}, tens digit of 2525^{2525}, and the hundreds digit of 2020^{200}?

    (A) 6
    (B) 8
    (C) 7
    (D) 13
    (E) NOTA

5. Find the next number in the pattern: 8, 13, 19, 26, 35, 47

    (A) 78
    (B) 62
    (C) 64
    (D) 61
    (E) NOTA

6. 1x1x1 cubes are put together to form a cube with side length 13, but such that the middle 5 blocks on every edge are missing, as well as a 2x2 square in the middle of every face. It is dunked in a vat of paint, and the cube is split apart again. What is the sum of the number of cubes that have 0 sides painted, 1 side painted, 2 sides painted, and 3 sides painted?

    (A) 2097
    (B) 2113
    (C) 1937
    (D) 2013
    (E) NOTA

7. What is $(1 + i)^5$?

    (A) $4 + 4i$
    (B) -4
    (C) 4
    (D) $-4 - 4i$
    (E) NOTA

8. The number of possible arrangements of the word AREA is equal to the number of indistinguishable pieces of paper I have in my pocket right now. How many ways are there to put those pieces of paper into 4 distinguishable buckets?

    (A) 495
    (B) 1820
    (C) 455
    (D) 720
    (E) NOTA

9. If $x + \dfrac{1}{x} = 5$, then what does $x^3 + \dfrac{1}{x^3}$ equal?

    (A) 125
    (B) 110
    (C) 123
    (D) 120
    (E) NOTA

10. A square is drawn with side length x. This square is divided into 4 smaller squares, and 3 of the 4 are shaded, with the non shaded smaller square posing as the new square. This process is done again. If the total shaded area is equal to 120, what is x?

    (A) $-4\sqrt{10}$ (B) 12   (C) $4\sqrt{10}$ (D) -12 (E) NOTA

11. Subtract the number of factors of 324 from the number of factors of 2020.

    (A) 3   (B) 2   (C) -2 (D) 6   (E) NOTA

12. Solve for x - 2y:

$$5x - 6y = -35$$
$$3x + 4y = 55$$

    (A) 5   (B) 10   (C) -15 (D) -10 (E) NOTA

13. The perpendicular bisector of the line segment with endpoints (5,6) and (1, 8) can be written in the form of $ax + by - c = 0$, with a, b, and c being the roots of a cubic equation in the form of $dx^3 + ex^2 + fx + $ g. What is e?

    (A) 1   (B) 0   (C) -3 (D) 2   (E) NOTA

14. What is $\frac{3}{5}$ of $\frac{3}{4}$ of 50% of 1000?

    (A) 500   (B) 125   (C) 375   (D) 225   (E) NOTA

15. Mihir has a 100 gallon vat of an extremely delicious solution of 85% liquid lucky charms. He wants to make a slightly less delicious solution of 50% liquid lucky charms. How many gallons of 10% liquid lucky charms does he need to add to make the new solution?

    (A) 92.5   (B) 96.5   (C) 87.5   (D) 82.5   (E) NOTA

16. How many different ways can 5 people be arranged around a table if two people have to sit next to each other?

    (A) 48   (B) 6   (C) 24   (D) 12   (E) NOTA

Questions 17-19 use the following lines:

$$X : y = 3x$$
$$Z : y = -3x + 12$$

17. Suppose the two equations of a line listed above intersect at point B. If the y-coordinate of point B can be written as the fraction $\frac{m}{n}$, where $m = 24$, what is n?

    (A) 6   (B) 4   (C) 12   (D) -3 (E) NOTA

18. Find the area of the shape formed by the x-axis and the region under both lines at point B.

    (A) 24   (B) 12   (C) 6   (D) 16π (E) NOTA

19. The line segment Akash can be formed by taking Line X and segmenting it from the origin to point B. The midpoint of that line can be written in the form (a,b). Find b^2 - a.

    (A) 8   (B) 4   (C) 5   (D) 2   (E) NOTA

20. If a triangle has two side lengths measuring 12 and 17 units, which of the following is not a possible length of its third side?

    (A) 16   (B) 25   (C) 8   (D) 21   (E) NOTA

21. Consider the following set:

    $$10, 20, 30, 40, 50 \ ... \ 186, 350$$

    What is the range of the data set?

    (A) $93, 175$ (B) $186, 340$ (C) 10   (D) $93, 180$ (E) NOTA

22. Consider the function $18x^2 + 39x + 20$. Another form to write it is $(Ax + B)(Cx + D)$. Find the following:

    $$AB + CD + AC + BD + AD + BC$$

    (A) 154   (B) 77   (C) 39   (D) 119   (E) NOTA

23. If Sina is 10% shorter than a 70 inch tall Erin, how much taller (in inches) is Erin compared to Sina?

    (A) 70   (B) 60   (C) 7   (D) 10   (E) NOTA

24. If $4x - 5y = 2$. Find the value of $\dfrac{81^x}{243^y}$.

    (A) $\dfrac{1}{3}$ (B) 9   (C) 3   (D) $\dfrac{2}{5}$ (E) NOTA

25. Consider the following expression:

    $$\frac{1}{2}x^2 - 12$$

    The expression written above can be written in the form $\dfrac{1}{2}(x + a)(x - a)$. What is the value of a?

    (A) $2\sqrt{3}$ (B) 12   (C) $2\sqrt{6}$ (D) 24   (E) NOTA

26. The question below uses the following system of equations:

$$x + 2y - 3z = 4$$
$$4x - 3y + 2z = 8$$
$$2x + 2y + 2z = 12$$

The solutions to this system can be written as $x = a$, $y = b$, and $z = c$ (where a, b, and c are unique solutions). Any combination of three of these is a three digit number (i.e. abc could be 999 but not 9 x 9 x 9). With those constraints in mind, find:

$$2000 \text{ - abc + cba}$$

(A) 1556   (B) 1978   (C) 1802   (D) 2020   (E) NOTA

27. Simplify the following:

$$\frac{(13! - 12!)}{(6! - 5!)}$$

(A) $\dfrac{13}{6}$ (B) 120 x 42 x 99 x $\dfrac{96}{5}$ (C) 13!   (D) 1   (E) NOTA

28. Sina owes Tanusri 10 dollars worth of starbucks coffee. After not paying her back for 6 months, Tanusri decides to make a deal and pay for the coffee herself, as long as he can solve the following problem:

Which of the following is equivalent to i^{768}?

Solve the problem for Sina!

(A) 1   (B) i (C) -1 (D) $-i$ (E) NOTA

29. Find the discriminant of the following quadratic equation:

$$3x^2 + 9x + 4 = 0$$

(A) 5   (B) 16   (C) 33   (D) 66   (E) NOTA

30. What is $\dfrac{1}{2}$ of $\dfrac{1}{2}$ of $\dfrac{1}{2}$ of $\dfrac{1}{2}$ of $\dfrac{1}{2}$ of $\dfrac{1}{2}$ of $\dfrac{1}{2}$?

(A) 0.0156625   (B) $\dfrac{1}{125}$ (C) 128   (D) 2   (E) NOTA